

Neutrosophic Relations Database

A. A. Salama¹, Mohamed Eisa² and M. M. Abdelmoghy³

^{1,3}Department of Mathematics and Computer Science, Faculty of Sciences, Port Said University Egypt.

drsalama44@gmail.com

²Computer Science Department, Port Said University, 42526 Port Said, Egypt.

mmmeisa@yahoo.com

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Abstract

In this paper we introduce and study new types of neutrosophic concepts " α -cut levels, normal neutrosophic set, convex neutrosophic set". Added to we investigate a neutrosophic relations, and neutrosophic database (NDB). Finally we study some neutrosophic queries on a neutrosophic database.

Keywords: Neutrosophic Set; Neutrosophic Relations; Neutrosophic Database.

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1. Introduction

The fuzzy set was introduced by Zadeh [20] in 1965, where each element had a degree of membership. The intuitionistic fuzzy set (Ifs for short) on a universe X was introduced by K. Atanassov [1, 2, 3] as a generalization of fuzzy set, where besides the degree of membership and the degree of non-membership of each element. After the introduction of the neutrosophic set concept [4, 6, 7, 8, 9, 10, 11]. The fundamental concepts of neutrosophic set, introduced by Smarandache in [9, 10] and Salama at el. in [4, 6, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18], provides a natural foundation for treating mathematically the neutrosophic phenomena which exist pervasively in our real world and for building new branches of neutrosophic mathematics. Neutrosophy has laid the foundation for a whole family of new mathematical theories generalizing both their classical and fuzzy counterparts [1, 2, 3, 4, 5, 19, 20], such as a neutrosophic set theory. In this paper, we will begin with a definition of neutrosophic relation and then define the various operations and will study its main properties. Intuitionistic fuzzy sets are neutrosophic sets but the converse is not true. An intuitionistic fuzzy database introduced by

Supriya at el. [19] is a generalization of fuzzy database. We here generalize intuitionistic fuzzy database by incorporating neutrosophic tolerance relation .

2. Terminologies

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [9, 10], Atanassov in [1, 2, 3], Salama [10, 11, 12, 13, 14, 15, 16, 17, 18] and Burillo at el [5].

Smarandache introduced the neutrosophic components T, I, F which represent the membership,

indeterminacy, and non-membership values respectively, where $[0,1]^{[-,+]}$ is non-standard unit interval.

2.1 Definition [9, 10]

Let T, I, F be real standard or nonstandard subsets of $[0,1]^{[-,+]}$, with

$$\text{Sup_T} = t_{\text{sup}}, \text{inf_T} = t_{\text{inf}}$$

$$\text{Sup_I} = i_{\text{sup}}, \text{inf_I} = i_{\text{inf}}$$

$$\text{Sup_F} = f_{\text{sup}}, \text{inf_F} = f_{\text{inf}}$$

$$n\text{-sup} = t_{\text{sup}} + i_{\text{sup}} + f_{\text{sup}}$$

$$n\text{-inf} = t_{\text{inf}} + i_{\text{inf}} + f_{\text{inf}},$$

T, I, F are called neutrosophic components

2.2 Definition [6, 10, 11]

Let X be a non-empty fixed set. A neutrosophic set (NS for short or $((A \in N^X))$) A is an object having the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X\}$ Where $\mu_A(x)$, $\sigma_A(x)$ and $\gamma_A(x)$ which represent the degree of membership function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$), and the degree of non-membership (namely $\gamma_A(x)$) respectively of each element $x \in X$ to the set A .

2.3 Definition [11]

The NSs 0_N and 1_N in X as follows:

0_N may be defined as:

$$(0_1) \quad 0_N = \{\langle x, 0, 0, 1 \rangle : x \in X\}$$

$$(0_2) \quad 0_N = \{\langle x, 0, 1, 1 \rangle : x \in X\}$$

$$(0_3) \quad 0_N = \{\langle x, 0, 1, 0 \rangle : x \in X\}$$

$$(0_4) \quad 0_N = \{\langle x, 0, 0, 0 \rangle : x \in X\}$$

1_N may be defined as:

$$(1_1) \quad 1_N = \{\langle x, 1, 0, 0 \rangle : x \in X\}$$

$$(1_2) \quad 1_N = \{\langle x, 1, 0, 1 \rangle : x \in X\}$$

$$(1_3) \quad 1_N = \{\langle x, 1, 1, 0 \rangle : x \in X\}$$

$$(1_4) \quad 1_N = \{\langle x, 1, 1, 1 \rangle : x \in X\}$$

3. α -Levels for Neutrosophic Sets

We must first introduce the concept of α -cut levels for neutrosophic sets

3.1 Definition

Let $A = \langle \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$ be a neutrosophic set of the set X . For $\alpha \in [0,1]$, the α -cut of A is the crisp set A_α defined by as two type :

Type 1. $A_\alpha = \{x : x \in X, \text{either } \mu_A(x), \sigma_A(x) \geq \alpha \text{ or } \nu_A(x) \leq 1-\alpha\}, \alpha \in \left[0,1\right]^- \cup \left[0,1\right]^+$ or

Type 2. $A_\alpha = \{x : x \in X, \text{either } \mu_A(x) \geq \alpha, \sigma_A(x) \leq \alpha \text{ or } \nu_A(x) \leq 1-\alpha\}, \alpha \in \left[0,1\right]^- \cup \left[0,1\right]^+$

it may condition $\mu_A(x) \geq \alpha$ ensures $\nu_A(x) \leq 1-\alpha$ but not conversely. So, we can define α -cut of A as $A_\alpha = \{x : x \in X, \nu_A(x) \leq 1-\alpha\}$.

3.2 Definition

For a neutrosophic set $A = \langle \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$ the weak α -cut defined as two types

Type 1. $A_\alpha = \{x : x \in X, \text{either } \mu_A(x), \sigma_A(x) > \alpha \text{ or } \nu_A(x) < 1-\alpha\}, \alpha \in \left[0,1\right]^- \cup \left[0,1\right]^+$ or

Type 2. $A_\alpha = \{x : x \in X, \text{either } \mu_A(x) > \sigma_A(x) < \alpha \text{ or } \nu_A(x) < 1-\alpha\}, \alpha \in \left[0,1\right]^- \cup \left[0,1\right]^+$

The strong α -cut defined as two types

Type 1. $A_{\alpha^-} = \{x : x \in X, \text{either } \mu_A(x), \sigma_A(x) \geq \alpha \text{ or } \nu_A(x) \leq 1-\alpha\}, \alpha \in \left[0,1\right]^- \cup \left[0,1\right]^+$ or

Type 2. $A_{\alpha^+} = \{x : x \in X, \text{either } \mu_A(x) \geq \sigma_A(x) \leq \alpha \text{ or } \nu_A(x) \leq 1-\alpha\}, \alpha \in \left[0,1\right]^- \cup \left[0,1\right]^+$

3.3 Definition

A neutrosophic set with $\mu_A(x) = 1, \sigma_A(x) = 1, \gamma_A(x) = 1$ is called normal neutrosophic set. In other words A is called normal neutrosophic set if and only if

$$\max_{x \in X} \mu_A(x) = \max_{x \in X} \sigma_A(x) = \max_{x \in X} \gamma_A(x) = 1.$$

3.4 Definition

When the support set is a real number set and the following applies for all $x \in [a,b]$ over any interval $[a,b]$: $\mu_A(x) \geq \mu_A(a) \wedge \mu_A(b)$; $\sigma_A(x) \geq \sigma_A(a) \wedge \sigma_A(b)$ and $\gamma_A(x) \geq \gamma_A(a) \wedge \gamma_A(b)$, A is said to be neutrosophic convex.

3.5 Definition

When $A \subset X$ and $B \subset Y$, the neutrosophic subset $A \times B$ of $X \times Y$ that can be arrived at the following way is the direct product of A and B .

$$A \times B \leftrightarrow \mu_{A \times B}(x, y) = \mu_A(x) \wedge \mu_B(y)$$

$$\sigma_{A \times B}(x, y) = \sigma_A(x) \wedge \sigma_B(y)$$

$$\gamma_{A \times B}(x, y) = \gamma_A(x) \wedge \gamma_B(y)$$

Making use α -cut, the following relational equation is called the resolution principle.

3.1 Theorem

$$\mu_A(x) = \gamma_A(x) = \sigma_A(x) = \sup_{x \in \left[0, 1\right]^{-+}} [\alpha \wedge \chi_{A_\alpha}(x)]$$

$$\mu_A(x) = \sigma_A(x) = \gamma_A(x) = \sup [\alpha \wedge \chi_{A_\alpha}(x)]$$

Proof

Clear

$$\text{The resolution principle is expressed in the form } A = \bigcup_{\alpha \in \left[0, 1\right]^{-+}} \alpha A_\alpha$$

In other words, a neutrosophic set can be expressed in terms of the concept of α -cuts without resorting to grade functions μ , δ and γ .

This is what wakes up the representation theorem, and we will leave it at that α -cuts are very convenient for the calculation of the operations and relations equations of neutrosophic sets.

In the next section we introduce the concept of neutrosophic database .

4 Neutrosophic Relations

Let X, Y and Z are ordinary finite non-empty sets.

4.1 Definition

We will call neutrosophic relation R from set X to set Y (or between X and Y) is a neutrosophic set in the direct product $X \times Y = \{(x, y) : x \in X, y \in Y\}$, that is, every neutrosophic sub sets of $X \times Y$ that is, to every expression R given $R = \{(x, y), (\mu_R(x, y), \sigma_R(x, y), \gamma_R(x, y)) : x \in X, y \in Y\}$ where $X \times Y$ is characterized by membership function $\mu_R(x)$, the degree of indeterminacy $\sigma_R(x)$, and the degree of non-membership $\gamma_R(x)$ respectively of each element $x \in X, y \in Y$ to the set X and Y. where $\mu_R : X \times Y \rightarrow \left[0, 1\right]^{-+}$, $\sigma_R : X \times Y \rightarrow \left[0, 1\right]^{-+}$, and $\gamma_R : X \times Y \rightarrow \left[0, 1\right]^{-+}$.

Given sets $X = \{x_1, x_2, \dots, x_m\}$, $Y = \{y_1, y_2, \dots, y_n\}$, a neutrosophic relation in $X \times Y$ can be expresses by an $m \times n$ matrix like the one in Fig 3.1. This kind of matrix, which expressed of neutrosophic relation, is called a neutrosophic matrix. Since the triple $(\mu_R, \sigma_R, \gamma_R)$ has values with in the interval $\left[0, 1\right]^{-+}$, the elements of the neutrosophic matrix also have values within $\left[0, 1\right]^{-+}$. In order to express neutrosophic relation R for $(\mu_R(x_i, y_i), \sigma_R(x_i, y_i), \gamma_R(x_i, y_i))$. The neutrosophic relation is defined as neutrosophic subsets of $X \times Y$, having the form $R = \{(x, y), (\mu_R(x, y), \sigma_R(x, y), \gamma_R(x, y)) : x \in X, y \in Y\}$. Where the triple $(\mu_R, \sigma_R, \gamma_R)$ has

values with in the interval $\left[\frac{-}{0,1} \frac{+}{\frac{1}{0}} \right]$, the elements of the neutrosophic matrix also have values with in $\left[\frac{-}{0,1} \frac{+}{\frac{1}{0}} \right]$.

4.2 Definition

Given a neutrosophic relation between X and Y we can define R^{-1} between Y and X by means of $\mu_{R^{-1}}(y, x) = \mu_R(x, y), \sigma_{R^{-1}}(y, x) = \sigma_R(x, y), \gamma_{R^{-1}}(y, x) = \gamma_R(x, y) \forall (x, y) \in X \times Y$. to which are will call inverse neutrosophic relation of R.

4.1 Example

When a neutrosophic relation R on X= {a, b, c} is

$$R = \langle (x, y), (0.2, 0.4, 0.3)(a, a), (1, 0.2, 0)(a, b), (0.4, 0.1, 0.7)(a, c), (0.6, 0.2, 0.1)(b, b), (0.3, 0.2, 0.6)(b, c), (0.2, 0.4, 0.1)(c, c) \rangle$$

The neutrosophic matrix for R is as shown: $R = \begin{bmatrix} < 0.2, 0.4, 0.3 > & < 1, 0.2, 0 > & < 0.4, 0.1, 0.7 > \\ < 0.6, 0.2, 0.1 > & < 0.3, 0.2, 0.6 > & < 0.3, 0.2, 0.6 > \end{bmatrix}$

Fig. 3.1

4.2 Example

Let X be a real number set. For $x, y \in X$, and the neutrosophic relation R can be characterized by the following:

$$\mu_R(x, y) = \begin{cases} 0 & ; x \geq y \\ \frac{1}{1 + \left(\frac{10}{y-x} \right)^2} & ; x < y \end{cases}$$

$$\sigma_R(x, y) = \begin{cases} 0 & ; x \geq y \\ \frac{1}{1 + \left(\frac{2}{y-x} \right)^2} & ; x < y \end{cases}$$

$$\gamma_R(x, y) = \begin{cases} 0 & ; x \geq y \\ \frac{1}{1 + \left(\frac{2}{y-x} \right)^2} & ; x < y \end{cases}$$

As a generalization of neutrosophic relations, the n-array neutrosophic relation R in $X_1 \times X_2 \times X_3 \times \dots \times X_n$ is given by

$$R = \prod_{X_1 \times X_2 \times X_3 \times \dots \times X_n} (\mu_R(x_1, x_2, \dots, x_n), \sigma_R(x_1, x_2, \dots, x_n), \gamma_R(x_1, x_2, \dots, x_n)) , \quad x_i \in X \text{ and we get the following:}$$

$$\mu_R : X_1 \times X_2 \times \cdots \times X_n \rightarrow \left[\begin{array}{c} - \\ 0,1 \\ + \end{array} \right]$$

$$\sigma_R : X_1 \times X_2 \times \cdots \times X_n \rightarrow \left[\begin{array}{c} - \\ 0,1 \\ + \end{array} \right]$$

$$\gamma_R : X_1 \times X_2 \times \cdots \times X_n \rightarrow \left[\begin{array}{c} - \\ 0,1 \\ + \end{array} \right]$$

When $n=1$, R is a unary neutrosophic relation, and this is clearly a neutrosophic set in X . when $n=2$, we have the relations of this paper. Other ways of expressing neutrosophic relations include matrices

We can define the operations of neutrosophic relations

4.3 Definition

Let R and S be two neutrosophic relations between X and Y for every $(x, y) \in X \times Y$ we can define

1) $R \subseteq S$ may be defined as two types

i) **Type 1:** $R \subseteq S \Leftrightarrow \mu_R(x, y) \leq \mu_S(x, y), \sigma_R(x, y) \leq \sigma_S(x, y), \gamma_R(x, y) \geq \gamma_S(x, y)$

ii) **Type 2:** $R \subseteq S \Leftrightarrow \mu_R(x, y) \leq \mu_S(x, y), \sigma_R(x, y) \geq \sigma_S(x, y), \gamma_R(x, y) \geq \gamma_S(x, y)$

2) $R \cup S$ may be defined as two types

i) **Type 1:** $R \cup S = \{\langle x, y \rangle, (\mu_R(x, y) \vee \mu_S(x, y), \sigma_R(x, y) \vee \sigma_S(x, y), \gamma_R(x, y) \wedge \gamma_S(x, y))\}$

ii) **Type 2:** $R \cup S = \{\langle x, y \rangle, (\mu_R(x, y) \vee \mu_S(x, y), \sigma_R(x, y) \wedge \sigma_S(x, y), \gamma_R(x, y) \wedge \gamma_S(x, y))\}$

3) $R \cap S$ may be defined as types:

i) **Type 1:** $R \cap S = \{\langle x, y \rangle, (\mu_R(x, y) \wedge \mu_S(x, y), \sigma_R(x, y) \wedge \sigma_S(x, y), \gamma_R(x, y) \vee \gamma_S(x, y))\}$

ii) **Type 2:** $R \cap S = \{\langle x, y \rangle, (\mu_R(x, y) \wedge \mu_S(x, y), \sigma_R(x, y) \vee \sigma_S(x, y), \gamma_R(x, y) \vee \gamma_S(x, y))\}$

(4) The complement of neutrosophic relation R (R^c for short) may be defined as three types:

i) **Type 1:** $R^c = \{(x, y), (\mu_R(x, y), \sigma_R(x, y), \gamma_R(x, y))\}$

ii) **Type 2:** $R^c = \{(x, y), (\gamma_R(x, y), \sigma_R(x, y), \mu_R(x, y))\}$

iii) **Type 3:** $R^c = \{(x, y), (\gamma_R(x, y), \sigma_R(x, y), \mu_R(x, y))\}$

4.1 Theorem

Let R , S and Q be three neutrosophic relations on $N(X \times Y)$ then

i) $R \subseteq S \Rightarrow R^{-1} \subseteq S^{-1}$,

ii) $(R \cup S)^{-1} = R^{-1} \cup S^{-1}$,

iii) $(R \cap S)^{-1} = R^{-1} \cap S^{-1}$

iv) $(R^{-1})^{-1} = R$.

- v) $R \cap (S \cup Q) = (R \cap S) \cup (R \cap Q)$.
- vi) $R \cup (S \cap Q) = (R \cup S) \cap (R \cup Q)$.
- vii) If $S \subseteq R, Q \subseteq R$, then $S \vee Q \subseteq R$.
- viii) If $R \subseteq S, R \subseteq Q$, then $R \leq S \cap Q$.

Proof

- i) If $R \subseteq S$ then $\mu_{R^{-1}}(y, x) = \mu_R(x, y) \leq \mu_S(x, y) = \mu_{S^{-1}}(y, x)$, for every (x, y) of $X \times Y$. Analogously $\sigma_{R^{-1}}(y, x) = \sigma_R(x, y) \geq \sigma_S(x, y)$ or $\leq \sigma_S(x, y)$ and $\gamma_{R^{-1}}(y, x) = \gamma_R(x, y) \geq \gamma_S(x, y) = \gamma_{S^{-1}}(y, x)$ for every (x, y) of $X \times Y$.
- ii) $\mu_{(R \vee S)^{-1}}(y, x) = \mu_{R \vee S}(x, y) = \mu_R(x, y) \vee \mu_S(x, y) = \mu_{R^{-1} \vee S^{-1}}(y, x) = \mu_{R^{-1}}(y, x) \vee \mu_{S^{-1}}(y, x)$. The proof for $\sigma_{(R \cup S)^{-1}}(y, x) = \sigma_{R^{-1} \cup S^{-1}}(x, y)$ and $\gamma_{(R \cup S)^{-1}}(y, x) = \gamma_{R^{-1} \cup S^{-1}}(x, y)$ done in a similar way. v), vii), and viii) clear from the definition of the operators \wedge and \vee .

4.4 Definition

- (1) The neutrosophic relation $I \in NR(X \times X)$ is called relation of identity , if

$$\begin{aligned}\mu_I(x, y) &= \begin{cases} 1 & \text{if } x=y \\ 0 & \text{if } x \neq y \end{cases} \quad \forall (x, y) \in X \times X \\ \sigma_I(x, y) &= \begin{cases} 1 & \text{if } x=y \\ 0 & \text{if } x \neq y \end{cases} \quad \forall (x, y) \in X \times X \text{ or } \sigma_I(x, y) = \begin{cases} 0 & \text{if } x=y \\ 1 & \text{if } x \neq y \end{cases} \quad \forall (x, y) \in X \times X \\ \gamma_I(x, y) &= \begin{cases} 0 & \text{if } x=y \\ 1 & \text{if } x \neq y \end{cases} \quad \forall (x, y) \in X \times X\end{aligned}$$

Will be represented by the symbol $I = I^{-1}$.

- (2) The complementary neutrosophic relation I^c defined by

$$\begin{aligned}\mu_{I^c}(x, y) &= \begin{cases} 0 & \text{if } x=y \\ 1 & \text{if } x \neq y \end{cases} \quad \forall (x, y) \in X \times X \\ \sigma_{I^c}(x, y) &= \begin{cases} 0 & \text{if } x=y \\ 1 & \text{if } x \neq y \end{cases} \quad \forall (x, y) \in X \times X \text{ or } \sigma_{I^c}(x, y) = \begin{cases} 1 & \text{if } x=y \\ 0 & \text{if } x \neq y \end{cases} \text{ or } \sigma_{I^c}(x, y) = \begin{cases} 0 & \text{if } x=y \\ 1 & \text{if } x \neq y \end{cases} \\ \forall (x, y) \in X \times X\end{aligned}$$

$$\gamma_{I^c}(x, y) = \begin{cases} 1 & \text{if } x=y \\ 0 & \text{if } x \neq y \end{cases} \quad \forall (x, y) \in X \times X \text{ Note that } I^c = (I^c)^{-1}.$$

We can defined some types of neutrosophic relations

4.5 Defintion

- The neutrosophic relation $R \in NR(X \times X)$ is called neutrosophic

- (1) Reflexive if for every $x \in X$, $\mu_R(x, x)=1$, and $\sigma_R(x, x)=0$ or $\sigma_R(x, x)=1$

Just notice $\gamma_R(x, x) = 0 \quad \forall x \in X$.

(2) Anti-reflexive neutrosophic relation if for every $x \in X$,

$$\begin{cases} \mu_R(x, x) = 0 \\ \sigma_R(x, x) = 0, \text{ or } \sigma_R(x, x) = 1 \\ \gamma_R(x, x) = 1 \end{cases}$$

4.2 Theorem

Let R be a reflexive neutrosophic relation in $X \times X$. Then

- (1) R_1^{-1} is reflexive neutrosophic relation.
- (2) $R_1 \cup R_2$ is reflexive neutrosophic relation for every $R_2 \in NR(X \times X)$.
- (3) $R_1 \cap R_2$ is reflexive neutrosophic relation $\Leftrightarrow R_2 \in NR(X \times X)$ is reflexive neutrosophic relation

Proof

Clear

Just notice that

- 1) $\mu_{R_1 \cup R_2}(x, x) = \mu_{R_1}(x, x) \vee \mu_{R_2}(x, x) = 1 \vee \mu_{R_2}(x, x) = 1$
- 2) $\sigma_{R_1 \cup R_2}(x, x) = \sigma_{R_1}(x, x) \vee \sigma_{R_2}(x, x) = 1 \vee \sigma_{R_2}(x, x) = 1 \text{ or } = 0 \vee \sigma_{R_2}(x, x) = \sigma_{R_2}(x, x)$
- 3) $\mu_{R_1 \cap R_2}(x, x) = \mu_{R_1}(x, x) \wedge \mu_{R_2}(x, x) = 1 \wedge \mu_{R_2}(x, x) = \mu_{R_2}(x, x)$
 $\sigma_{R_1 \cap R_2}(x, x) = \sigma_{R_1}(x, x) \wedge \sigma_{R_2}(x, x) = 1 \wedge \sigma_{R_2}(x, x) = \sigma_{R_2}(x, x)$
 $\text{or } = 0 \wedge \sigma_{R_2}(x, x) = 0$

4.6 Definition

- (1) The neutrosophic relation $R \in NR(X \times X)$ is called symmetric if $R = R^{-1}$, that is for every $(x, y) \in X \times Y$

$$\begin{cases} \mu_R(x, y) = \mu_R(y, x) \\ \sigma_R(x, y) = \sigma_R(y, x) \\ \gamma_R(x, y) = \gamma_R(y, x) \end{cases}$$

- (2) The neutrosophic relation $R \in NR(X \times X)$ we will say that it is anti-symmetrical neutrosophic relation if $\forall (x, y) \in X \times Y$, The definition of anti-symmetrical neutrosophic relation is justified because of the following argument $x \leq_R y$ if and only if is an order the referential X if the neutrosophic relation $R \in NR(X \times X)$ is reflexive and anti-symmetrical.

4.3 Theorem

Let $R \in NR(X \times X)$. R is anti-symmetrical neutrosophic relation if and only if $\forall (x, y) \in X \times Y$, with $x \neq y$ then $\mu_R(x, y) \neq \mu_R(y, x)$

Proof

As $\gamma_R(x, y) = \mu^c R(x, y) \forall (x, y) \in X \times Y$, then $\mu_R(x, y) \neq \mu_R(y, x)$ if and only if

$$\begin{cases} \mu_R(x, y) \neq \mu_R(y, x) \\ \sigma_R(x, y) \neq \sigma_R(y, x) \text{ or } \sigma_R(x, y) = \sigma_R(y, x) \\ \gamma_R(x, y) \neq \gamma_R(y, x) \end{cases}$$

4.7 Definition

Let $R \in NR(X \times X)$, we will call transitive neutrosophic closure of R to the minimum neutrosophic relation T on $X \times X$ which contains R and it is transitive, that is to say

- i) $R \subseteq T$
- ii) If $R, P \in N(X, X)$, $R \subseteq P$ and P is transitive, then $T \subseteq P$.

4.4 Theorem

Let $R, P, T, S \in NR(X \times X)$ and $R \subseteq P$ and $R \subseteq T$, $R \subseteq S$, then $T \subseteq S$.

Proof

Clear from Definitions.

4.8 Definition

If R is a neutrosophic relation in $X \times Y$ and S is a neutrosophic relation in $Y \times Z$, the composition of R and S , $R \circ S$ is a neutrosophic relation in $X \times Z$ as defined below

$$1- R \circ S \leftrightarrow (R \circ S)(x, z) = \left\{ \vee_y (\mu_R(x, y) \wedge \mu_S(y, z)), \vee_y (\sigma_R(x, y) \wedge \sigma_S(y, z)), \vee_y (\gamma_R(x, y) \wedge \gamma_S(y, z)) \right\}$$

$$2- R \circ S \leftrightarrow (R \circ S)(x, z) = \left\{ \wedge_y (\mu_R(x, y) \vee \mu_S(y, z)), \wedge_y (\sigma_R(x, y) \vee \sigma_S(y, z)), \wedge_y (\gamma_R(x, y) \vee \gamma_S(y, z)) \right\}$$

4.9 Definition

A neutrosophic relation R on the cartesian set $X \times X$, is called

- i) A neutrosophic tolerance relation on $X \times X$ if R is reflexive and symmetric
- ii) A neutrosophic similarity (equivalence) relation on $X \times X$ if R is reflexive, symmetric and Transitive

4.1 Example

Consider the neutrosophic tolerance relation T on $X = \{x_1, x_2, x_3, x_4\}$ given

	x_1	x_2	x_3	x_4
x_1	$<1,0,0>$	$<0.8,0.2,0.1>$	$<0.6,0.1,0.2>$	$<0.3,0.3,0.4>$
x_2	$<0.8,0.2,0.1>$	$<1,0,0>$	$<0.4,0.4,0.5>$	$<0.5,0.2,0.3>$
x_3	$<0.6,0.1,0.2>$	$<0.4,0.4,0.5>$	$<1,0,0>$	$<0.6,0.2,0.3>$
x_4	$<0.3,0.3,0.4>$	$<0.5,0.2,0.3>$	$<0.6,0.2,0.3>$	$<1,0,0>$

It can be computed that for $\alpha = 1$, the partition of X determined by T_α given

by $\{\{x_1\}, \{x_2\}, \{x_3\}, \{x_4\}\}$, for $\alpha = 0.9$, the partition of X determined by T_α given by $\{\{x_1, x_2\}, \{x_3\}, \{x_4\}\}$,

$\alpha = 0.8$, the partition of X determined by T_α given by $\{\{x_1, x_2, x_3\}, \{x_4\}\}$,

$\alpha = 0.7$, the partition of X determined by T_α given by $\{\{x_1, x_2, x_3, x_4\}\}$,

Moreover, we see that when $\alpha \in (0.9, 1]$ the partition of X determined by T_α given

by $\{\{x_1\}, \{x_2\}, \{x_3\}, \{x_4\}\}$,

when $\alpha \in (0.8, 0.9]$, the partition of X determined by T_α given by $\{\{x_1, x_2, x_3\}, \{x_4\}\}$,

when $\alpha \in (0.7, 0.8]$, the partition of X determined by T_α given by $\{\{x_1, x_2, x_3\}, \{x_4\}\}$,
when $\alpha \in (0, 0.7]$, the partition of X determined by T_α given by $\{\{x_1, x_2, x_3\}, \{x_4\}\}$,

In the next section we introduce the concept of neutrosophic database

5. Neutrosophic Database

5.1 Definition

A neutrosophic database relation R is a subset of cross product $2^{D_1} \times 2^{D_2} \times \dots \times 2^{D_m}$, where

$$2^{D_j} = 2^{D_1} - \phi$$

5.2 Definition

Let $R \subseteq 2^{D_1} \times 2^{D_2} \times \dots \times 2^{D_m}$, be a neutrosophic database relation. A neutrosophic set tuple (with respect to R) is an element of R.

Let $t_i = (d_{i_1}, d_{i_2}, \dots, d_{i_m})$ be a neutrosophic tuple. An interpolation of t_i is a tuple $\theta = (a_1, a_2, \dots, a_m)$ where $a_j \in d_{ij}$ for each domain D_j . for each domain D_j , if T_j be the neutrosophic tolerance relation then the membership function is given by $\mu_{T_j} : D_j \times D_j \rightarrow [0,1]$, the non-membership function is given by $\gamma_{T_j} : D_j \times D_j \rightarrow [0,1]$ and indeterminacy $\sigma_{T_j} : D_j \times D_j \rightarrow [0,1]$.

Let us make a hypothetical case study below:

We consider a criminal data file. Suppose that one murder has taken place at some area in deem light. The police suspects that the murderer is also from the same area and so police refer to a data file of all the suspected criminals of the that area. Listening to the eye-witness, the police has discovered that the criminal for that murder case has more or less or non-more and less curly hair texture and he has moderately large build. Form the criminal data file, the information table with attributes " Hair Coverage", Hair Texture", and "Build" is given by

Name	Hair Coverage	Hair Texture	Build
Soso	Full Small (FS)	Stc	Large
Toto	Rec.	Wavy	Very Small(VS)
Koko	Full Small(FS)	Straight(Str.)	Small(S)
Momo	Bald	Curly	Average(A)
Wowo	Bald	Wavy	Average(A)
Bobo	Full Big (FB)	Stc.	Very large(VL)
Hoho	Full Small	Straight	Small(S)
Vovo	Rec.	Curly	Average(A)

Now, consider the Neutrosophic Tolerance Relation T_{D_1} where $D_1 = \text{"Hair Coverage"}$, which is given by:

	FB	FS	Rec.	Bald
FB	$<1,0,0>$	$<0.8,0.3,0.1>$	$<0.4,0,0.4>$	$<0,0.2,1>$
FS	$<0.8,0.3,0.1>$	$<1,0,0>$	$<0.5,0,0.4>$	$<0,0,0.9>$

Rec.	<0.4,0,0.4>	<0.5,0,0.4>	<1,0,0>	<0.4,0,0.4>
Bald	<0,0.2,1>	<0,0,0.9>	<0.4,0,0.4>	<1,0,0>

Where, Hair Coverage= {FB,FS,Rec.,Bald}.

The Neutrosophic Tolerance Relation T_{D_2} where D_2 = "Hair Texture" is given by:

	Str.	Stc.	Wavy	Curly
Str.	<1,0,0>	<0.7,0.2,0.3>	<0.2,0.2,0.7>	<0.1,0.2,0.7>
Stc.	<0.7,0.2,0.3>	<1,0,0>	<0.3,0,0.4>	<0.5,0,0.2>
Wavy	<0.2,0.2,0.7>	<0.3,0,0.4>	<1,0,0>	<0.4,0,0.4>
Bald	<0.1,0.2,0.7>	<0.5,0,0.2>	<0.4,0,0.4>	<1,0,0>

Where, Hair Texture= {Str., Stc., Wavy, Curly }.

Also, Neutrosophic Tolerance Relation T_{D_3} where D_3 = "Build" is given by:

	Vl	L	A	S	Vs
Vl	<1,0,0>	<0.8,0,0.2>	<0.5,0,0.4>	<0.3,0,0.6>	<0,1,1>
L	<0.8,0,0.2>	<1,0,0>	<0.6,0,0.4>	<0.4,0,0.5>	<0,0,0.9>
A	<0.5,0,0.4>	<0.6,0,0.4>	<1,0,0>	<0.6,0,0.3>	<0.3,0,0.6>
S	<0.3,0,0.6>	<0.4,0,0.5>	<0.5,0,0.4>	<1,0,0>	<0.8,0,0.2>
Vs	<0,1,1>	<0,0,0.9>	<0.3,0,0.6>	<0.8,0,0.2>	<1,0,0>

Where, Build = { Vl, L, A, S, Vs }.

Now, the job is to find out a list of those criminals who resemble with more or less or non big hair coverage with more or less or non curly hair texture and moderately large build. This list will be useful to the police for further investigation. It can be translated into relational algebra in the following form:

Project (Select (CRIMINALS DATA FILE)

Where HAIR COVERAGE="FULL BIG",

HAIR TEXRURE="CURLY"

BUILLD="LARG"

With $\alpha - LEVEL(HAIR COVERAGE) = 0.8$

$\alpha - LEVEL(HAIR TEXRURE) = 0.8$

$\alpha - LEVEL(BUILLD) = 0.7$

With $\alpha - LEVEL(NAME) = 0.0$

With $\alpha - LEVEL(HAIR COVERAGE) = 0.8$

$\alpha - LEVEL(HAIR TEXRURE) = 0.8$

$\alpha - LEVEL(BUILLD) = 0.7$

gining LIKELY MURDERER)

Result: It can be computed that the above neutrosophic query gives rise to the following relation:
LIKELY MURDERER

NAME	HAIR COVERAGE	HAIR TEXRURE	BUILLD
{SOSO, BOBO}	{FULL BIG, FULL SMALL}	{CURLY, STC.}	{LARG, VERY LARG}

Therefore, according to the information obtained from the eye-witness, police concludes that Soso or Bobo are the likely murderers. and, further investigation now is to be done on them only, instead of dealing with huge list of criminals.

CONCLUSION

Neutrosophic set theory takes care of such indeterministic part in connection with each references point of its universe. In the presnt paper we have introduced a concept of neutrosophic database (NDB) and have shown by an example the usefulness of neutrosophic queries on a neutrosophic database.

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